EYFS

Addition

Objective & strategy	Concrete	Pictorial	Abstract
One more than a number	Use everyday objects, Cubes and counters to find one more than any given number to 20. Build a Numicon track and do a walk of one more.	Use of pictorial representations to count one more than a number. One more than 3 is 4. Use of a number track and a counter or whiteboard pen to count on a jump of one more than.	Use of mental maths to count on from the biggest number one more. $8 ... 9$ $8 + 1 = 9$
Adding Two Single Digit Numbers:	Use everyday objects, cubes and counters to add. Children will start by counting all beginning with 1 when children are secure can move them on to counting on from one number 5,6,7,8,9,10,11. Use Numicon to add single digits. Children start by counting all circles, later on	Children draw pictures and use representations of pictures to count all or count on from the biggest number.	Use of a numbered line to count on from the biggest number. $5 + 5 = 10$
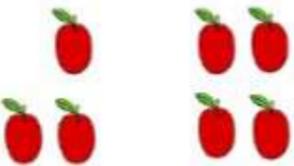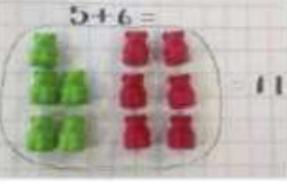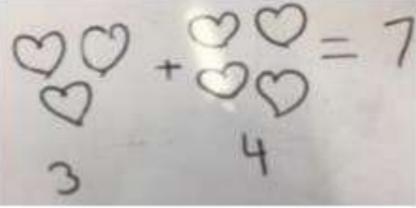
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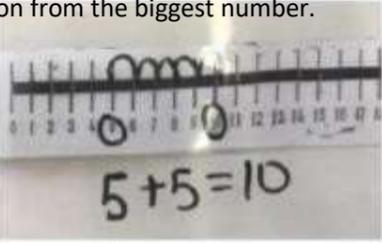
	recognising the shape and counting on.		

Aberford C of E Primary School – CPA Calculations Policy 20/21

Year 1

Addition

Objective & Strategy	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part-whole model	Use part part whole model. Use cubes to add two numbers together as a group or in a bar.	Use pictures to add two numbers together as a group or in a bar.	$4 + 3 = 7$ $10 = 6 + 4$ Use the part-part whole diagram as shown above to move into the abstract.
Starting at the bigger number and counting on	Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.	$12 + 5 = 17$ Start at the larger number on the number line and count on in ones or in one jump to find the answer.	$5 + 12 = 17$ Place the larger number in your head and count on the smaller number to find your answer.
Regrouping to make 10. *This is an essential skill for column addition later.*	$6 + 5 = 11$ Start with the bigger number and use the smaller number to make 10. Use ten frames.	$3 + 9 =$ Use pictures or a number line. Regroup or partition the smaller number using the part part whole model to make 10. $9 + 5 = 14$	$7 + 4 = 11$ If I am at seven, how many more do I need to make 10. How many more do I add on now?
Represent & use number bonds and related subtraction facts within 20	2 more than 5.	$5 + 2 =$	Emphasis should be on the language '1 more than 5 is equal to 6.' '2 more than 5 is 7.' '8 is 3 more than 5.'
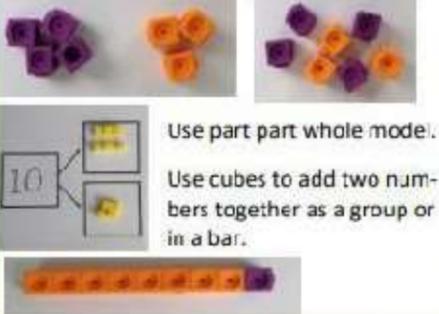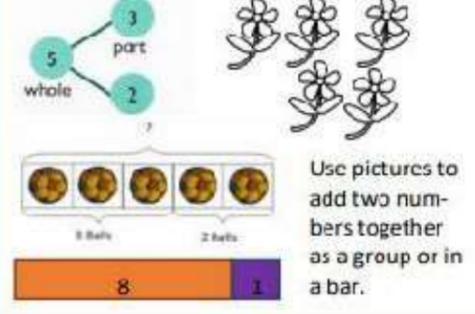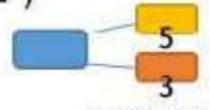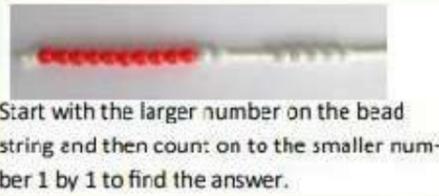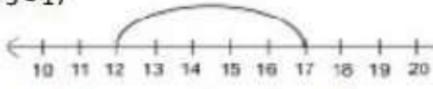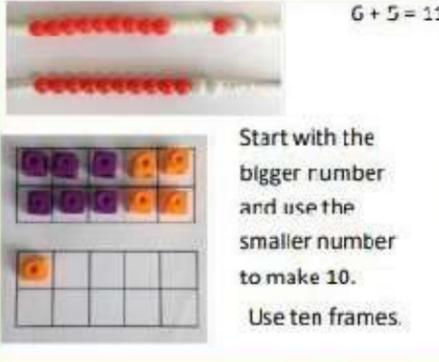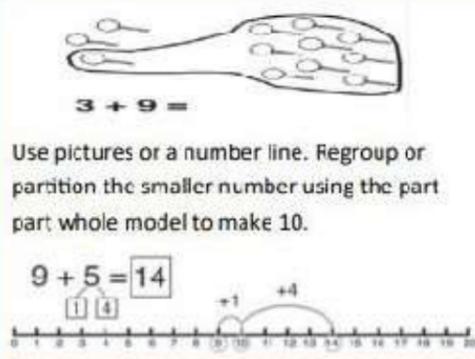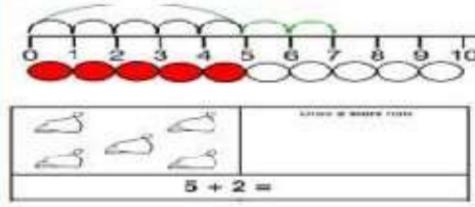
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Year 2

Addition

Objective & Strategy	Concrete	Pictorial	Abstract
Adding multiples of ten	50= 30 = 20 Model using dienes and bead strings	3 tens + 5 tens = _____ tens 30 – 50 = _____ Use representations for base ten.	20 + 30 = 50 70 = 50 + 20 40 + □ = 60
Use known number facts Part part whole	20 Children explore ways of making numbers within 20	20 ⟨ □ □ □ + □ = 20 20 – □ = □ □ + □ = 20 20 – □ = □	□ + 1 = 16 16 – 1 = □ 1 + □ = 16 16 – □ = 1
Using known facts	□□ + □□ = □□□□ ▯▯▯ + ▯▯▯ = ▯▯▯▯▯▯	∴ + ∴ = ∴ ||| + |||| = ||||||| ▪▪ + ▪▪ = ▪▪ Children draw representations of H,T and O	3 + 4 = 7 leads to 30 + 40 = 70 leads to 300 + 400 = 700
Bar model	3 + 4 = 7	7 + 3 = 10	| 23 | 25 | | ? | 23 + 25 = 48
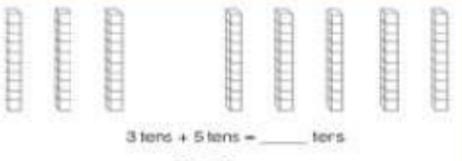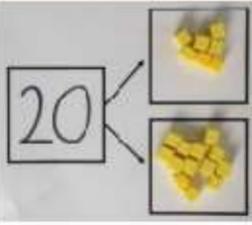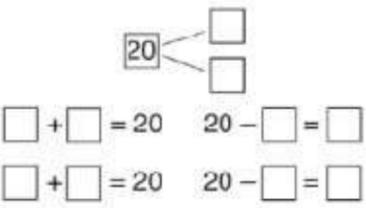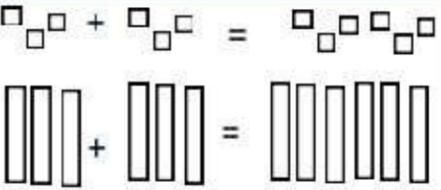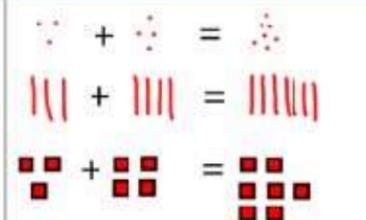
Year 2

Addition

Objective & Strategy	Concrete	Pictorial	Abstract
Add a two digit number and ones	$17 + 5 = 22$ Use ten frame to make 'magic ten' Children explore the pattern. $17 + 5 = 22$ $27 + 5 = 32$	$17 + 5 = 22$ Use part part whole and number line to model. 3 2 20 $16 + 7$ 16 20 23	$17 + 5 = 22$ Explore related facts $17 + 5 = 22$ $5 + 17 = 22$ $22 - 17 = 5$ $22 - 5 = 17$ 22 17 5
Add a 2 digit number and tens	$25 + 10 = 35$ Explore that the ones digit does not change	$27 + 30$ $+10$ $+10$ $+10$ 27 37 47 57	$27 + 10 = 37$ $27 + 20 = 47$ $27 + \square = 57$
Add two 2-digit numbers	Model using dienes, place value counters and numicon	$+20$ $+5$ Or $+20$ $+3$ $+2$ 47 67 72 47 67 70 72 Use number line and bridge ten using part whole if necessary.	$25 + 47$ $20 + 5$ $40 + 7$ $20 + 40 = 60$ $5 + 7 = 12$ $60 + 12 = 72$
Add three 1-digit numbers	Combine to make 10 first if possible, or bridge 10 then add third digit	Regroup and draw representation. $+$ $+$ $+$ $= 15$	$4 + 7 + 6 = 10 + 7$ 10 $= 17$ Combine the two numbers that make/bridge ten then add on the third.
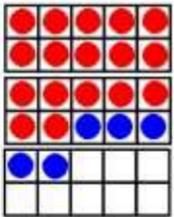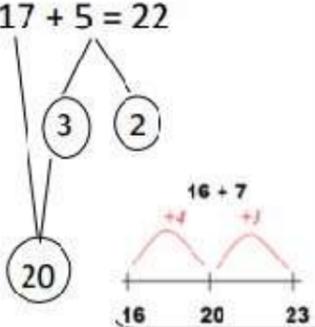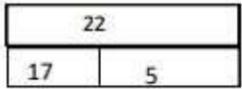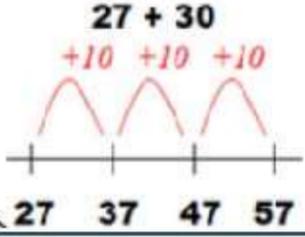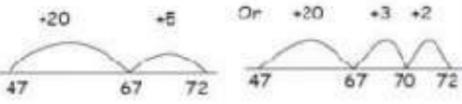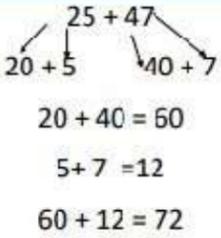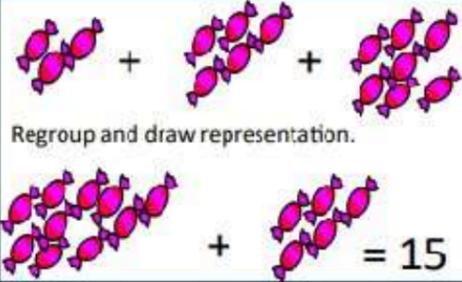
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Year 3

Addition

Objective & Strategy	Concrete	Pictorial	Abstract
Column Addition—no regrouping (friendly numbers) Add two or three 2 or 3-digit numbers.	Model using Dienes or numicon Add together the ones first, then the tens. 45 34 7 9 Move to using place value counters 21 + 42 = 21 + 42	Children move to drawing the counters using a tens and one frame. tens ones	2 2 3 + 1 1 4 3 3 7 Add the ones first, then the tens, then the hundreds.
Column Addition with regrouping.	39 15 5 4 Exchange ten ones for a ten. Model using numicon and pv counters. 146 + 527	Children can draw a representation of the grid to further support their understanding, carrying the ten **underneath** the line 5 1	$20 + 5$ $40 + 8$ $60 + 13 = 73$ Start by partitioning the numbers before formal column to show the exchange. 536 + 85 621 1 1
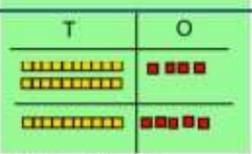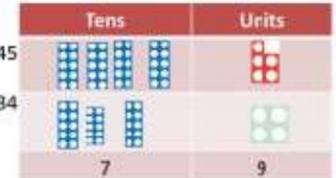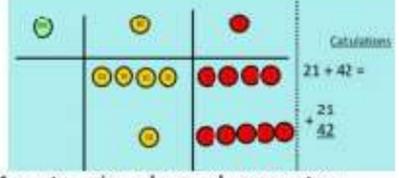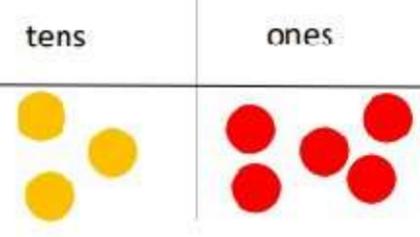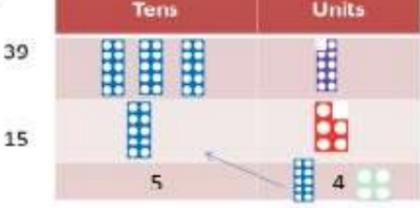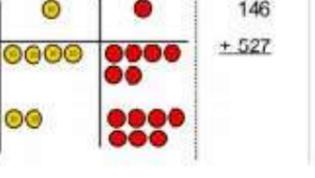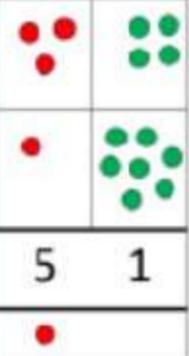
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Objective & Strategy	Concrete	Pictorial	Abstract
Y4—add numbers with up to 4 digits	Children continue to use dienes or pv counters to add, exchanging ten ones for a ten and ten tens for a hundred and ten hundreds for a thousand. Hundreds / Tens / Ones	7 1 5 1 Draw representations using pv grid.	$3517 + 396 = 3913$ Continue from previous work to carry hundreds as well as tens. Relate to money and measures.
Y5—add numbers with more than 4 digits. Add decimals with 2 decimal places, including money.	As year 4 tens ones • tenths hundredths Introduce decimal place value counters and model exchange for addition.	2.37 + 81.79 tens ones tenths hundredths	$72.8 + 54.6 = 127.4$ 1 1 £23.59 + £7.55 = £31.14
Y6—add several numbers of increasing complexity Including adding money, measure and decimals with different numbers of decimal points.	As Y5	As Y5	81,059 3,668 15,301 + 20,551 120,579 Insert zeros for place holders 23.361 9.080 59.770 + 1.300 93.511

Year 1

Subtraction

Objective & Strategy	Concrete	Pictorial	Abstract
Taking away ones.	Use physical objects, counters, cubes etc to show how objects can be taken away. $6-4=2$ $4-2=2$	$15 - 3 = \boxed{12}$ Cross out drawn objects to show what has been taken away.	$7-4=3$ $16-9=7$
Counting back	Move objects away from the group, counting backwards. Move the beads along the bead string as you count backwards.	$5 - 3 = 2$ Count back in ones using a number line.	Put 13 in your head, count back 4. What number are you at?
Find the Difference	Compare objects and amounts 7 'Seven is 3 more than four' 4 'I am 2 years older than my sister' 5 Pencils 3 Erasers ? Lay objects to represent bar model.	Count on using a number line to find the difference. +8	Hannah has 12 sweets and her sister has 5. How many more does Hannah have than her sister.?

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Year 1

Subtraction

Objective & Strategy	Concrete	Pictorial	Abstract
Represent and use number bonds and related subtraction facts within 20 Part Part Whole model	Link to addition. Use PPW model to model the inverse. If 10 is the whole and 6 is one of the parts, what s the other part? $10-6 = 4$	Use pictorial representations to show the part.	Move to using numbers within the part whole model. 12 — 5 — 7
Make 10	$14-9$ Make 14 on the ten frame. Take 4 away to make ten, then take one more away so that you have taken 5.	$13-7$ $13 - 7 = \boxed{6}$ Jump back 3 first, then another 4. Use ten as the stopping point.	$16-8$ How many do we take off first to get to 10? How many left to take off?
Bar model	$5-2 = 3$		8 — 2 $10 = 8 + 2$ $10 = 2 + 8$ $10-2 = 8$ $10-8 = 2$
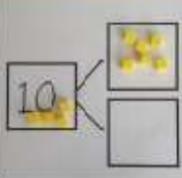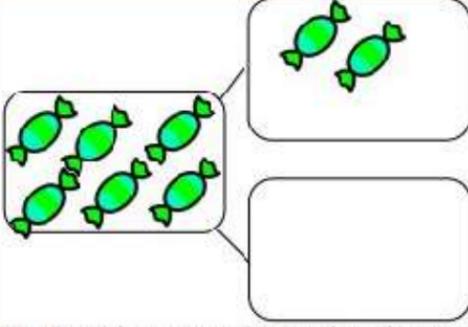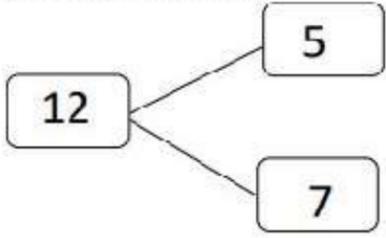
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Year 2

Subtraction

Objective & Strategy	Concrete	Pictorial	Abstract
Regroup a ten into ten ones	Use a PV chart to show how to change a ten into ten ones, use the term 'take and make'	$20 - 4 =$	$20 - 4 = 16$
Partitioning to subtract without regrouping. *'Friendly numbers'*	$34 - 13 = 21$ Use Dienes to show how to partition the number when subtracting without regrouping.	Children draw representations of Dienes and cross off. $43 - 21 = 22$	$43 - 21 = 22$
Make ten strategy *Progression should be crossing one ten, crossing more than one ten, crossing the hundreds.*	$34 - 28$ Use a bead bar or bead strings to model counting to next ten and the rest.	'counting on' to find 'difference' Use a number line to count on to next ten and then the rest.	$93 - 76 = 17$
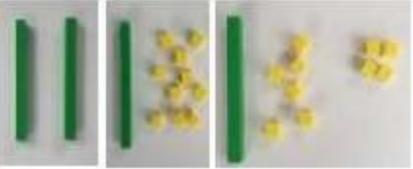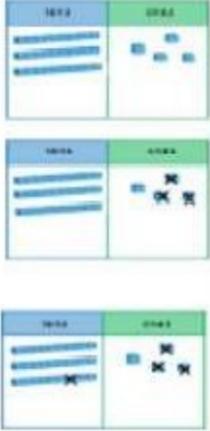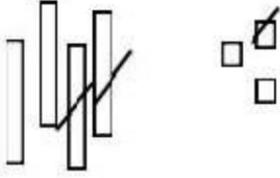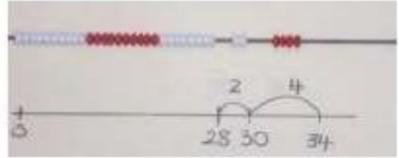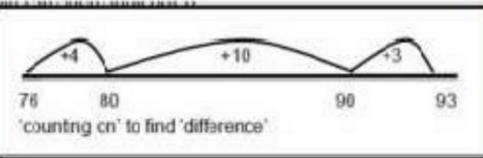
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Year 3

Subtraction

Objective & Strategy	Concrete	Pictorial	Abstract
Column subtraction without regrouping (friendly numbers)	$47-32$ Use base 10 or Numicon to model	Calculations $\begin{array}{r} 54 \\ -22 \\ \hline 32 \end{array}$ Draw representations to support understanding	$47-24=23$ $\begin{array}{r} 40+7 \\ -20+4 \\ \hline 20+3 \end{array}$ Intermediate step may be needed to lead to clear subtraction understanding. $\begin{array}{r} 32 \\ -12 \\ \hline 20 \end{array}$
Column subtraction with regrouping	**Tens Units** Begin with base 10 or Numicon. Move to pv counters, modelling the exchange of a ten into tten ones. Use the phrase 'take and make' for exchange.	$\begin{array}{r} 45 \\ -29 \\ \hline 16 \end{array}$ Tens Ones $= 16$ $10 + 6 = 16$ Children may draw base ten or PV counters and cross off.	$836-254=582$ $\begin{array}{rrr} 800 & 130 & 6 \\ 200 & 50 & 4 \\ \hline 500 & 80 & 2 \end{array}$ Begin by partitioning into pv columns $728-582=146$ $\begin{array}{rrr} ^6 7 & ^1 2 & 8 \\ 5 & 8 & 2 \\ \hline 1 & 4 & 6 \end{array}$ Then move to formal method.
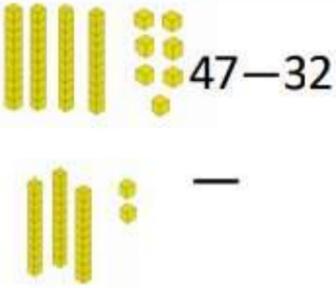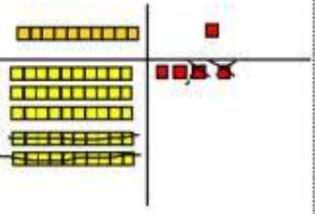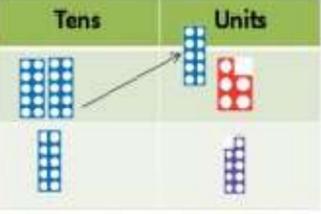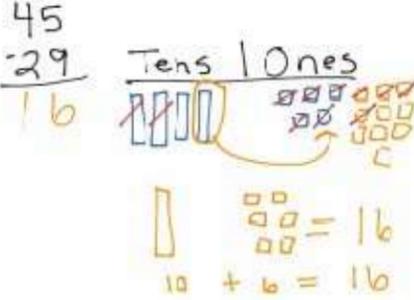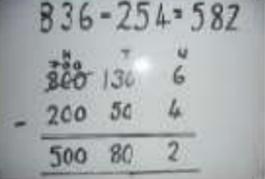
Aberford C of E Primary School – CPA Calculations Policy 20/21

Objective & Strategy	Concrete	Pictorial	Abstract
Subtracting tens and ones Year 4 subtract with up to 4 digits. *Introduce decimal subtraction through context of money*	234 - 179 Model process of exchange using Numicon, base ten and then move to PV counters.	Children to draw pv counters and show their exchange—see Y3	$$\begin{array}{r} 2\,\cancel{7}^{6}5\,4 \\ -\ 1\,5\,6\,2 \\ \hline 1\,1\,9\,2 \end{array}$$ Use the phrase 'take and make' for exchange
Year 5- Subtract with at least 4 digits, including money and measures. *Subtract with decimal values, including mixtures of integers and decimals and aligning the decimal*	As Year 4	Children to draw pv counters and show their exchange—see Y3	$$\begin{array}{r} 3\,1\,0\,8\,6 \\ -\quad 2\,1\,2\,8 \\ \hline 2\,8{,}9\,2\,8 \end{array}$$ Use zeros for place-holders. $$\begin{array}{r} 7\,1\,6\,8\,.\,0 \\ -\quad 3\,7\,2\,.\,5 \\ \hline 6\,7\,9\,6\,.\,5 \end{array}$$
Year 6—Subtract with increasingly large and more complex numbers and decimal values.			$$\begin{array}{r} 1\,8\,0{,}6\,9\,9 \\ -\quad 8\,9{,}9\,4\,9 \\ \hline 6\,0{,}7\,5\,0 \end{array}$$ $$\begin{array}{r} 1\,0\,5\,.\,4\,1\,9\ kg \\ -\quad 3\,6\,.\,0\,8\,0\ kg \\ \hline 6\,9\,.\,3\,3\,9\ kg \end{array}$$
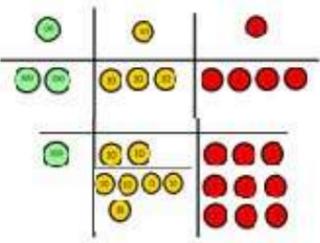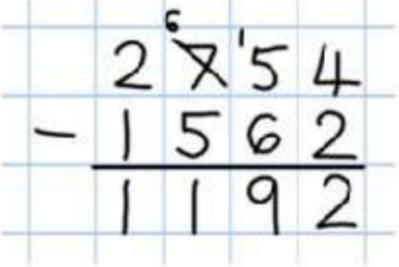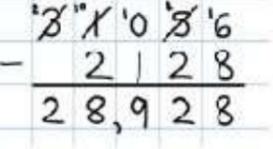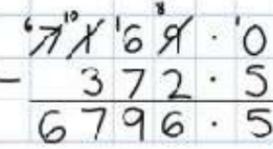
Year 4, 5, 6

Subtraction

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Aberford C of E Primary School – CPA Calculations Policy 20/21

Year 1

Multiplication

Objective & Strategy	Concrete	Pictorial	Abstract
Doubling	Use practical activities using manipulatives including cubes and Numicon to demonstrate doubling double 4 is 8 $4 \times 2 = 8$	Draw pictures to show how to double numbers Double 4 is 8	Partition a number and then double each part before recombining it back together. 16 10 6 $\times 2$ $\times 2$ $20 + 12 = 32$
Counting in multiples	Count the groups as children are skip counting, children may use their fingers as they are skip counting.	Children make representations to show counting in multiples.	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25 , 30
Making equal groups and counting the total	Use manipulatives to create equal groups. $\square \times \square = 8$	Draw ⊲◁ to show $2 \times 3 = 6$ Draw and make representations	$2 \times 4 = 8$

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Year 1

Multiplication

Objective & Strategy	Concrete	Pictorial	Abstract
Repeated addition	Use different objects to add equal groups	Use pictorial including number lines to solve prob There are 3 sweets in one bag. How many sweets are in 5 bags altogether? 3+3+3+3+3 = 15	Write addition sentences to describe objects and pictures. 2 + 2 + 2 + 2 + 2 = 10
Understanding arrays	Use objects laid out in arrays to find the answers to 2 lots 5, 3 lots of 2 etc.	Draw representations of arrays to show understanding	$3 \times 2 = 6$ $2 \times 5 = 10$
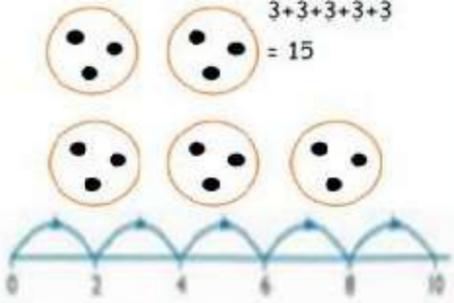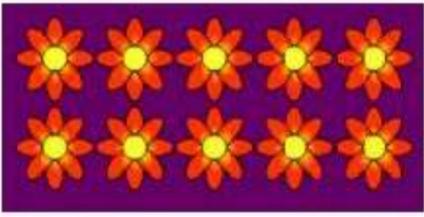
Year 2

Multiplication

Objective & Strategy	Concrete	Pictorial	Abstract
Doubling	Model doubling using dienes and PV counters. $40 + 12 = 52$	Draw pictures and representations to show how to double numbers	Partition a number and then double each part before recombining it back together. 16 10 6 $\times 2$ $\times 2$ $20 + 12 = 32$
Counting in multiples of 2, 3, 4, 5, 10 from 0 (repeated addition)	Count the groups as children are skip counting, children may use their fingers as they are skip counting. Use bar models. $5+5+5+5+5+5+5+5=40$	Number lines, counting sticks and bar models should be used to show representation of counting in multiples. 3 3 3 3 ?	Count in multiples of a number aloud. Write sequences with multiples of numbers. 0, 2, 4, 6, 8, 10 0, 3, 6, 9, 12, 15 0, 5, 10, 15, 20, 25, 30 $4 \times 3 = \square$
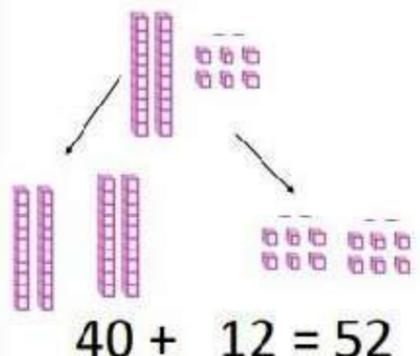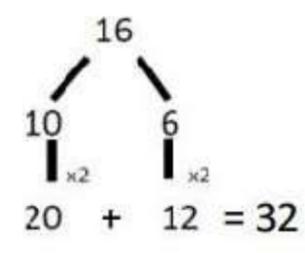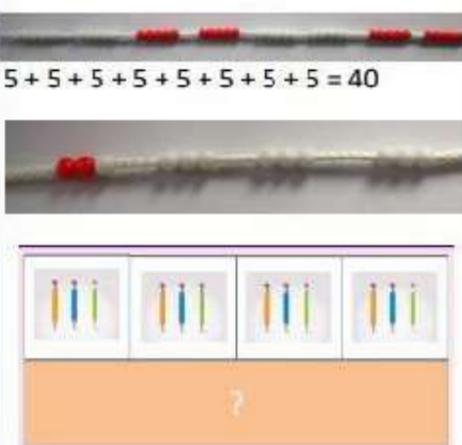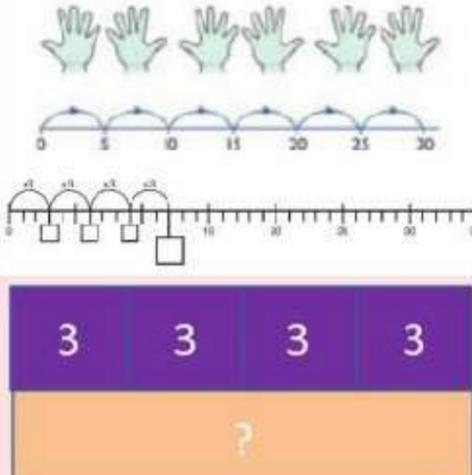
15

Objective & Strategy	Concrete	Pictorial	Abstract
Multiplication is commutative	Create arrays using counters and cubes and Numicon. Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.	Use representations of arrays to show different calculations and explore commutativity.	$12 = 3 \times 4$ $12 = 4 \times 3$ Use an array to write multiplication sentences and reinforce repeated addition. $5 + 5 + 5 = 15$ $3 + 3 + 3 + 3 + 3 = 15$ $5 \times 3 = 15$ $3 \times 5 = 15$
Using the Inverse *This should be taught alongside division, so pupils learn how they work alongside each other.*			$2 \times 4 = 8$ $4 \times 2 = 8$ $8 \div 2 = 4$ $8 \div 4 = 2$ $8 = 2 \times 4$ $8 = 4 \times 2$ $2 = 8 \div 4$ $4 = 8 \div 2$ Show all 8 related fact family sentences.
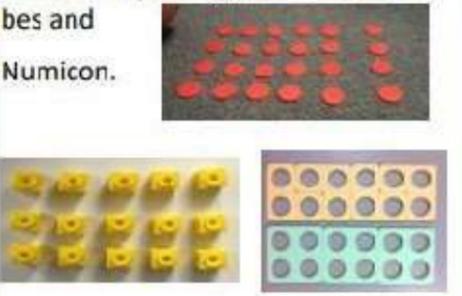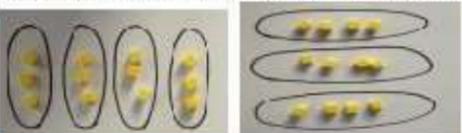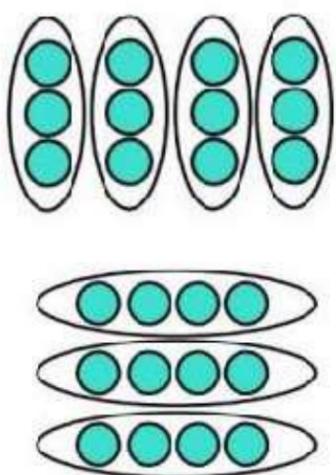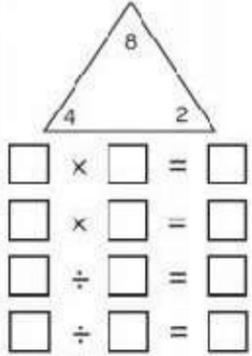
Year 3

Multiplication

Objective & Strategy	Concrete	Pictorial	Abstract
Grid method	Show the links with arrays to first introduce the grid method. 4 rows of 10 4 rows of 3 Move onto base ten to move towards a more compact method. 4 rows of 13 Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows Calculations 4 x 126 Fill each row with 126 Calculations 4 x 126 Add up each column, starting with the ones making any exchanges needed Then you have your answer.	Children can represent their work with place value counters in a way that they understand. They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below. Bar model are used to explore missing numbers 4 x ☐ = 20	Start with multiplying by one digit numbers and showing the clear addition alongside the grid. <table><tr><td>×</td><td>30</td><td>5</td></tr><tr><td>7</td><td>210</td><td>35</td></tr></table> 210 + 35 = 245 Moving forward, multiply by a 2 digit number showing the different rows within the grid method. <table><tr><td></td><td>10</td><td>8</td></tr><tr><td>10</td><td>100</td><td>80</td></tr><tr><td>3</td><td>30</td><td>24</td></tr></table>
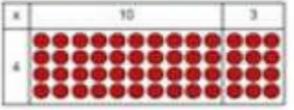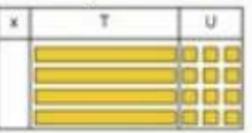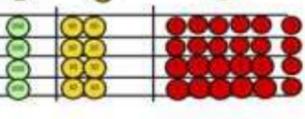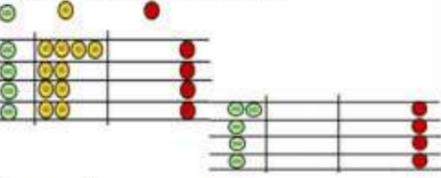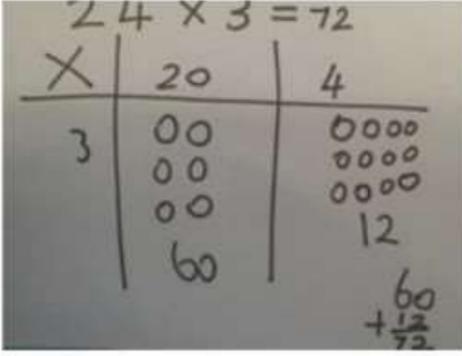
17

Objective & Strategy	Concrete	Pictorial	Abstract
Grid method recap from year 3 for 2 digits x 1 digit Move to multiplying 3 digit numbers by 1 digit. (year 4 expectation)	Use place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows Calculations 4 x 126 Fill each row with 126 Add up each column ... nes making any exchanges needed	Children can represent their work with place value counters in a way that they understand. They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below. 	Start with multiplying by one digit numbers and showing the clear addition alongside the grid. <table><tr><td>x</td><td>30</td><td>5</td></tr><tr><td>7</td><td>210</td><td>35</td></tr></table> 210 + 35 = 245
Column multiplication	Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. 321 x 2 = 642 It is important at this stage that they always multiply the ones first. The corresponding long multiplication is modelled alongside	<table><tr><td>x</td><td>300</td><td>20</td><td>7</td></tr><tr><td>4</td><td>1200</td><td>80</td><td>28</td></tr></table> The grid method my be used to show how this relates to a formal written method. Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.	327 x 4 ─── 28 80 1200 ──── 1308 This may lead to a compact method.
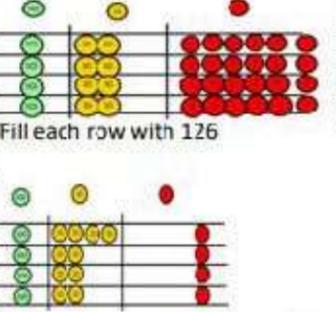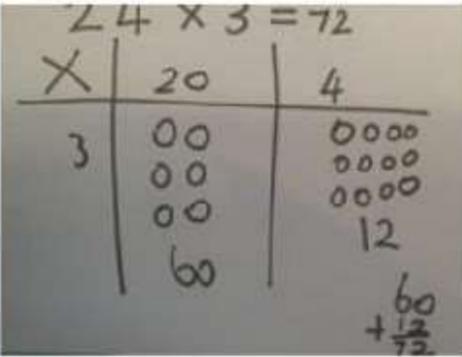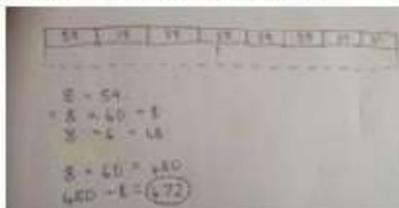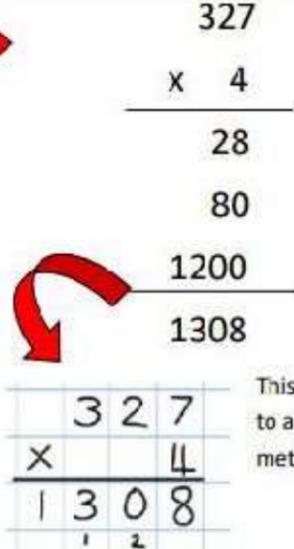
Year 5/6

Multiplication

Objective & Strategy	Concrete	Pictorial	Abstract
Column Multiplication for 3 and 4 digits x 1 digit.	It is important at this stage that they always multiply the ones first. Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. 321 x 2 = 642	 $$\begin{array}{c|c|c|c} \times & 300 & 20 & 7 \\ \hline 4 & 1200 & 80 & 28 \end{array}$$	327 x 4 28 80 1200 1308 327 x 4 1308 1 2 This will lead to a compact method.
Column multiplication	Manipulatives may still be used with the corresponding long multiplication modelled alongside.	 Continue to use bar modelling to support problem solving	 18 x 3 on the first row (8 x 3 =24, carrying the 2 for 20, then 1 x 3) 18 x 10 on the 2nd row. Show multiplying by 10 by putting zero in units first 1234 x 16 7404 (1234 x 6) 12340 (1234 x 10) 19,744

Year 6

Objective & Strategy	Concrete	Pictorial	Abstract
Multiplying decimals up to 2 decimal places by a single digit.			Remind children that the single digit belongs in the units column. Line up the decimal points in the question and the answer. $$\begin{array}{r} 3.19 \\ \times\ \ 8 \\ \hline 25.52 \end{array}$$

Multiplication

Year 1

Division

Objective & Strategy	Concrete	Pictorial	Abstract
Division as sharing *Use Gordon ITPs for modelling*	 I have 10 cubes, can you share them equally in 2 groups?	Children use pictures or shapes to share quantities. 8 shared between 2 is 4 Sharing: 4 4 4 12 shared between 3 is 4	12 shared between 3 is 4
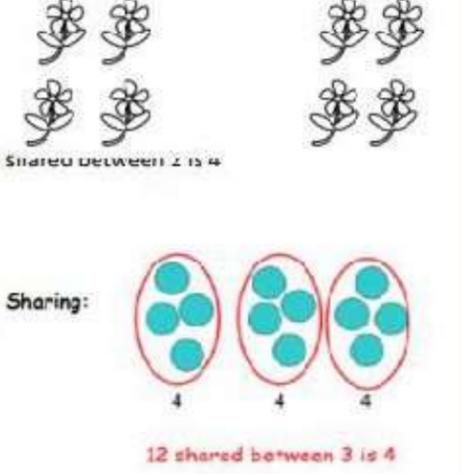
Year 2

Division

Objective & Strategy	Concrete	Pictorial	Abstract
Division as sharing	I have 10 cubes, can you share them equally in 2 groups?	Children use pictures or shapes to share quantities. $8 \div 2 = 4$ Children use bar modelling to show and support understanding. $12 \div 4 = 3$	$12 \div 3 = 4$
Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.	Use number lines for grouping $12 \div 3 = 4$ Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group. $20 \div 5 = ?$ $5 \times ? = 20$	$28 \div 7 = 4$ Divide 28 into 7 groups. How many are in each group?
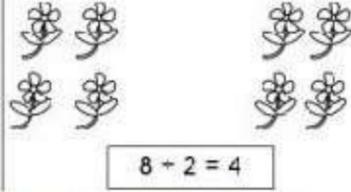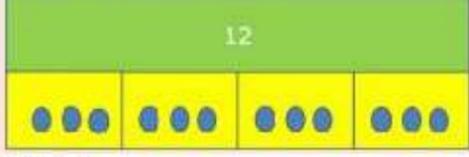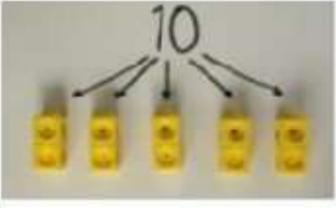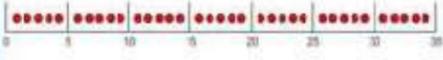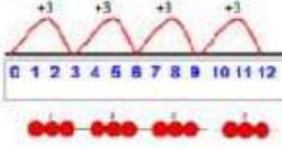
Year 3

Division

Objective & Strategy	Concrete	Pictorial	Abstract
Division as grouping	Use cubes, counters, objects or place value counters to aid understanding. 24 divided into groups of 5 – 4 $96 \div 3 = 32$	Continue to use bar modelling to aid solving division problems. 20 ? $20 \div 5 = ?$ $5 \times ? = 20$	How many groups of 6 in 24? $24 \div 6 = 4$
Division with arrays	Link division to multiplication by creating an array and thinking about the number sentences that can be created. Eg $15 \div 3 = 5$ $5 \times 3 = 15$ $15 \div 5 = 3$ $3 \times 5 = 15$	Draw an array and use lines to split the array into groups to make multiplication and division sentences	Find the inverse of multiplication and division sentences by creating eight linking number sentences. $7 \times 4 = 28$ $4 \times 7 = 28$ $28 \div 7 = 4$ $28 \div 4 = 7$ $28 = 7 \times 4$ $28 = 4 \times 7$ $4 = 28 \div 7$ $7 = 28 \div 4$
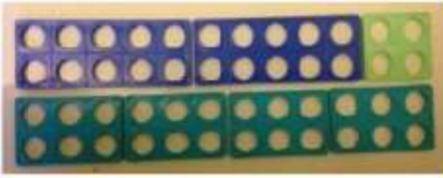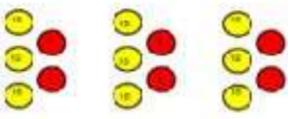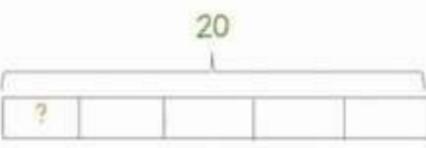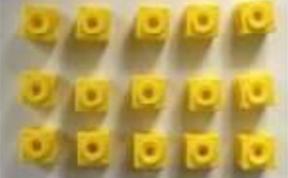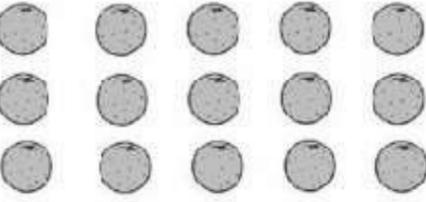
23

Year 3

Division

Objective & Strategy	Concrete	Pictorial	Abstract
Division with remainders.	$14 \div 3 =$ Divide objects between groups and see how much is left over 	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder. Draw dots and group them to divide an amount and clearly show a remainder. remainder 2 Use bar models to show division with remainders. Example without remainder: $40 \div 5$ Ask "How many 5s in 40?" $5+5+5+5+5+5+5+5$ = 8 fives 0 5 10 15 20 25 30 35 40 Example with remainder: $38 \div 6$ $6+6+6+6+6+6+2$ = 6 sixes with a remainder of 2 0 6 12 18 24 30 36 38 For larger numbers, when it becomes inefficient to count in single multiples, bigger jumps can be recorded using known facts.	Complete written divisions and show the remainder using r. $29 \div 8 = 3$ REMAINDER 5 dividend divisor quotient remainder

24

Objective & Strategy	Concrete	Pictorial	Abstract
Divide at least 3 digit numbers by 1 digit. Short Division	96 ÷ 3 **Tens** 3 **Units** 2 Use place value counters to divide using the bus stop method alongside Calculations 42 ÷ 3 42 ÷ 3= Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over. We exchange this ten for ten ones and then share the ones equally among the groups. We look how much in 1 group so the answer is 14.	Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups. Encourage them to move towards counting in multiples to divide more efficiently.	Begin with divisions that divide equally with no remainder. $\begin{array}{r} 2\ 1\ 8 \\ \hline 4\ )\ 8\ 7\ 2 \end{array}$ (³) Move onto divisions with a remainder. $\begin{array}{r} 8\ 6\ r\ 2 \\ \hline 5\ )\ 4\ 3\ 2 \end{array}$ (³) Finally move into decimal places to divide the total accurately. $\begin{array}{r} 1\ 4\ .\ 6 \\ \hline 3\ 5\ )\ 5\ 1\ 1\ .\ 0 \end{array}$ (16)(21) $\begin{array}{r} 0\ 6\ 6\ 3\ r\ 5 \\ \hline 8\ )\ 5\ ^5 3\ ^5 0\ ^2 9 \end{array}$

Year 4, 5, 6

Division

Long Division

Division

Step 1—a remainder in the ones

```
      h t o
    0 4 1 R1
4 ) 1 6 5
```

4 does not go into 1 (hundred). So combine the 1 hundred with the 6 tens (160).

4 goes into 16 four times.

4 goes into 5 once, leaving a remainder of 1.

```
    th h t o
   0 4 0 0 R7
8 ) 3 2 0 7
```

8 does not go into 3 of the thousands. So combine the 3 thousands with the 2 hundreds (3,200).

8 goes into 32 four times (3,200 ÷ 8 = 400)
8 goes into 0 zero times (tens).
8 goes into 7 zero times, and leaves a remainder of 7.

Long Division

Step 1 continued...

```
         h  t  o
         0  6  1
    4 ) 2  4  7
            - 4
               3
```

When dividing the ones, 4 goes into 7 one time. Multiply $1 \times 4 = 4$, write that four under the 7, and subtract. This finds us the remainder of 3.

Check: $4 \times 61 + 3 = 247$

```
       th  h  t  o
        0  4  0  2
    4 ) 1  6  0  9
                - 8
                   1
```

When dividing the ones, 4 goes into 9 two times. Multiply $2 \times 4 = 8$, write that eight under the 9, and subract. This finds us the remainder of 1.

Check: $4 \times 402 + 1 = 1,609$

Year 6

Division

27

Long Division

Year 6

Division

Step 2—a remainder in the tens

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
t o 2 2⟌5 8	t o 2 2⟌5 8 − 4 1	t o 2 9 2⟌5 8 − 4 ↓ 1 8
Two goes into 5 two times, or 5 tens ÷ 2 = 2 whole tens -- but there is a remainder!	To find it, multiply 2 × 2 = 4, write that 4 under the five, and subtract to find the remainder of 1 ten.	Next, drop down the 8 of the ones next to the leftover 1 ten. You combine the remainder ten with 8 ones, and get 18.

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
t o 2 9 2⟌5 8 − 4 1 8	t o 2 9 2⟌5 8 − 4 1 8 − 1 8 0	t o 2 9 2⟌5 8 − 4 1 8 − 1 8 0
Divide 2 into 18. Place 9 into the quotient.	Multiply 9 × 2 = 18, write that 18 under the 18, and subtract.	The division is over since there are no more digits in the dividend. The quotient is 29.

28

Long Division

Year 6

Division

Step 2—a remainder in any of the place values

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
h t o 1 2)278	h t o 1 2)278 -2 0	h t o 18 2)278 -2↓ 07
Two goes into 2 one time, or 2 hundreds ÷ 2 = 1 hundred.	Multiply 1 × 2 = 2, write that 2 under the two, and subtract to find the remainder of zero.	Next, drop down the 7 of the tens next to the zero.

Divide.	Multiply & subtract.	Drop down the next digit.
h t o 13 2)278 -2 07	h t o 13 2)278 -2 07 -6 1	h t o 13 2)278 -2 07 -6 18
Divide 2 into 7. Place 3 into the quotient.	Multiply 3 × 2 = 6, write that 6 under the 7, and subtract to find the remainder of 1 ten.	Next, drop down the 8 of the ones next to the 1 leftover ten.

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
h t o 139 2)278 -2 07 -6 18	h t o 139 2)278 -2 07 -6 18 -18 0	h t o 139 2)278 -2 07 -6 18 -18 0
Divide 2 into 18. Place 9 into the quotient.	Multiply 9 × 2 = 18, write that 18 under the 18, and subtract to find the remainder of zero.	There are no more digits to drop down. The quotient is 139.

Recognise, find and name a half as one of two equal parts of an object, shape or quantity.

Year 1

Concrete

$\frac{1}{2}$

Pictorial

A whole apple

Half an apple

I

$\frac{1}{2}$

Abstract

Half of 10 =

Half of 8 =

of 14 =

nise, one

$\frac{1}{2}$

Recog-find and name a quarter as four equal parts of an object, shape or quantity.

Concrete

$\frac{1}{4}$

Pictorial

$\frac{1}{4}$

Abstract
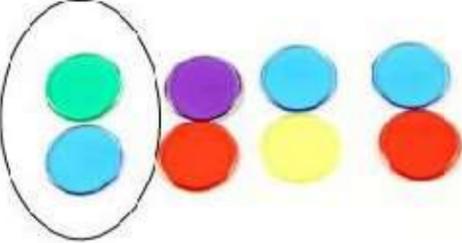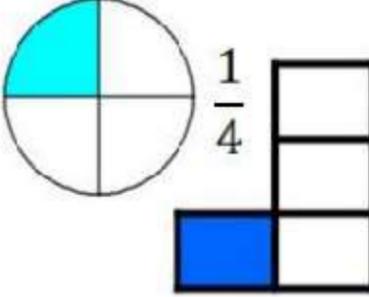
A quarter of 20 =

A quarter of 12 =

of 8 =

$\frac{1}{4}$

Fractions

30

Recognise, find and name and write fractions 1/3, 1/4, 2/4 and 3/4 of a length, shape, set of objects or quantity.

Year 2

Fractions

Concrete	Pictorial	Abstract
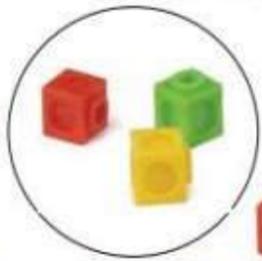
and recognise the 1/2.

Write simple equivalence
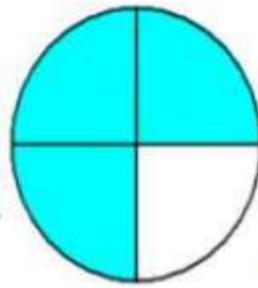
fractions of 2/4 and

$\frac{2}{4}$ of 8 = ☐
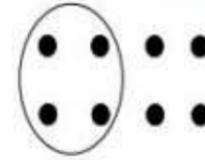
Concrete	Pictorial	Abstract
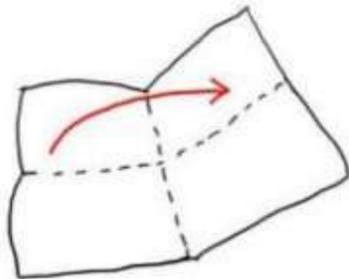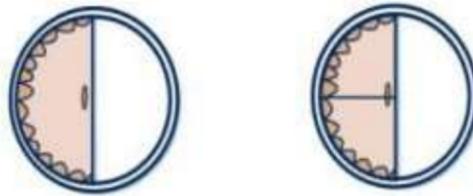
I have $\frac{1}{2}$ a pie   You have $\frac{2}{4}$ of a pie

$\frac{1}{2}$ of 6 = ☐
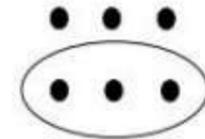
31

Count up and down in tenths: recognise that tenths arise from dividing an object into ten equal parts and in dividing one-digit numbers or quantities by ten.

Year 3

Fractions

Concrete

0   $\frac{1}{10}$ $\frac{2}{10}$ $\frac{3}{10}$ $\frac{4}{10}$ $\frac{5}{10}$ $\frac{6}{10}$ $\frac{7}{10}$ $\frac{8}{10}$ $\frac{9}{10}$ 1
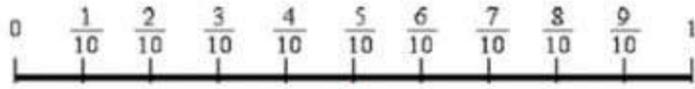
Recognise, find and write fractions of a discrete set of objects: unit fractions and non-unit fractions as numbers. $\frac{1}{2}$ fractions and use numbers.

Pictorial

$\frac{3}{10}$ $\frac{3}{10}$ $\frac{3}{10}$ a distractions frac-
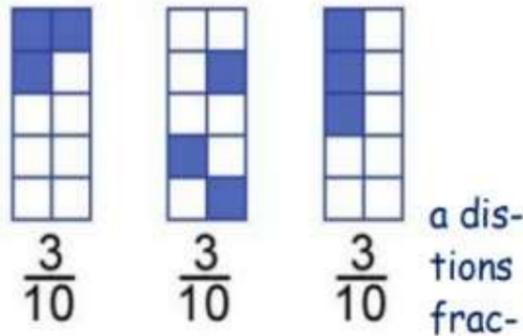
Abstract

$\frac{1}{10}$ of 6 = 0.6
because
6 ÷ 10 = 0.6

$\frac{1}{10}$ of 7 = 0.7
because
7 ÷ 10 = 0.7

Concrete

$\frac{1}{8}$

Pictorial

$\frac{3}{4}$
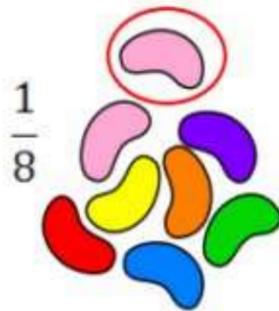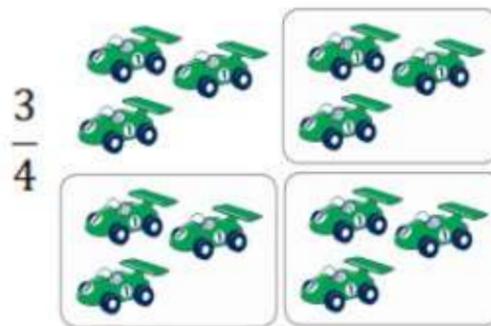
Abstract

$\frac{1}{5}$ of 15 sweets = 3
because 15 ÷ 5 = 3

$\frac{2}{5}$ of 15 sweets = 6
because 15 ÷ 5 = 3 and 3 x 2 = 6

Recognise and show, using diagrams, equivalent fractions with small denominators.

Concrete

two halves
$\frac{2}{2}$

four quarters
$\frac{4}{4}$
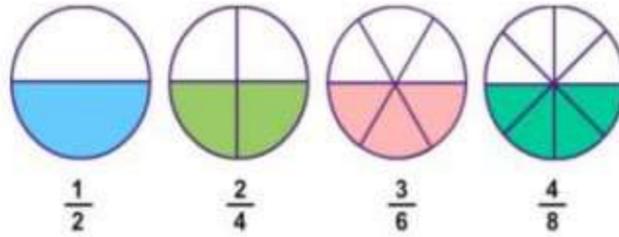
Pictorial

$\frac{1}{2}$ $\frac{2}{4}$ $\frac{3}{6}$ $\frac{4}{8}$

Abstract

Sam says that two quarters is the same as one half.

Is he correct?

How do you know?

Year 3

Add sub-tract fractions with the same denominator.

Concrete
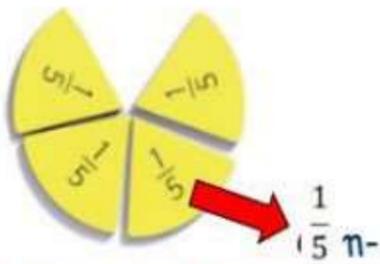
$\frac{1}{5}$ n-

and order unit fractions the same denominators.

Pictorial
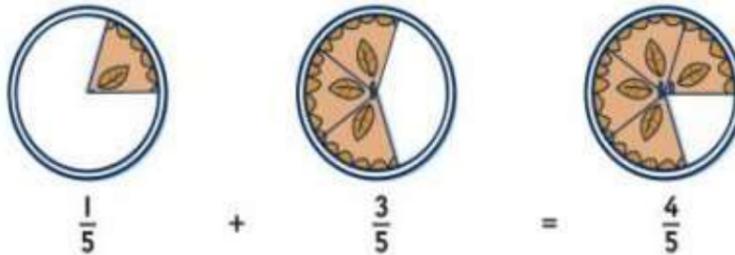
$\frac{1}{5}$ + $\frac{3}{5}$ = $\frac{4}{5}$

Abstract

$\frac{5}{7} + \frac{1}{7} = \frac{6}{7}$

$\frac{5}{8} - \frac{2}{8} = \frac{3}{8}$ pare with

and

Concrete

$\frac{1}{6}$ > $\frac{1}{6}$
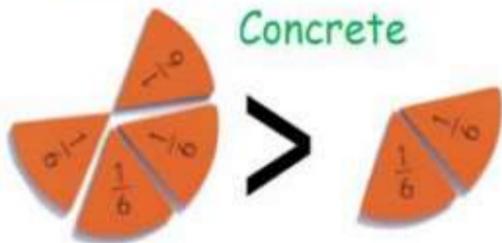
Pictorial

<
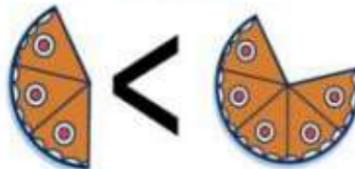
Abstract

$\frac{2}{8}$ $\frac{3}{8}$ $\frac{5}{8}$ $\frac{7}{8}$

Fractions

33

Count up and down in hundredths: recognise that hundredths arise when dividing an object by 100 and dividing tenths by 10.

Year 4

Concrete

0

0.1

0.00 0.01 0.02 0.03 0.04 0.05 0.06 0.07 0.08 0.09 0.10

Recog-lents to $\frac{3}{100}$ nise and $\frac{7}{100}$ write decimal 1/2, 1/4 and 3/4.

Pictorial

thousands | hundreds | tens | ones | Decimal Point | tenths | hundredths

1 hundredth = 0.01 = $\frac{1}{100}$

equiva-
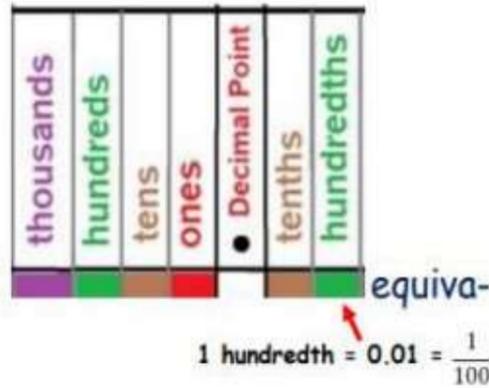
Abstract

$\frac{1}{100}$ of 60 = 0.6
because 60 ÷ 100 = 0.6

$\frac{1}{10}$ of 70 = 0.7
so $\frac{1}{100}$ of 70 = 0.07

Fractions

Concrete

0.5

0.25

1/2

1/4

Pictorial

$\frac{1}{4}$ = 0.25 $\frac{1}{2}$ = 0.5 $\frac{3}{4}$ = 0.75
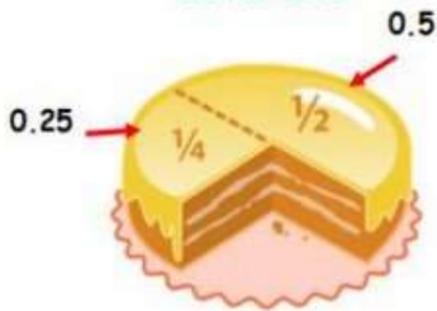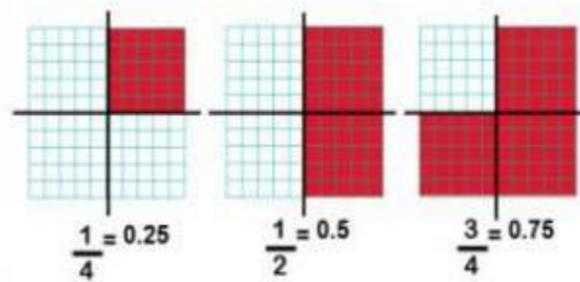
Abstract

$\frac{1}{2}$ = 0.5

$\frac{1}{4}$ = 0.25

$\frac{3}{4}$ = 0.75

34

Year 4

Fractions

Concrete

Rec- $\frac{1}{10}$ of the chocolate bar = 0.1

Pictorial

0.6
six tenths
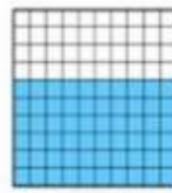
0.60
sixty hundredths

ognise grams, families of common equivalents.

Abstract

$\frac{1}{10} = 0.1$

$\frac{3}{10} = 0.3$

$\frac{5}{10} = \frac{1}{2} = 0.5$

$\frac{8}{100} = 0.08$

and show, using dia-

Concrete

$\frac{1}{3}$ $\frac{1}{3}$ = $\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$
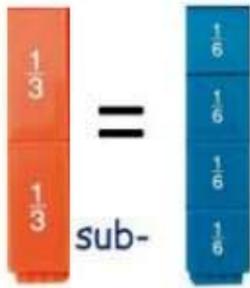
Add and sub- tract fractions
nator.
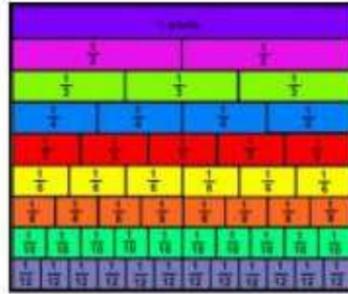
Pictorial

Abstract

$\frac{2}{3} = \frac{4}{6}$

$\frac{3}{5} = \frac{6}{10}$

$\frac{2}{12} = \frac{1}{6}$

with the same denomi-

Concrete
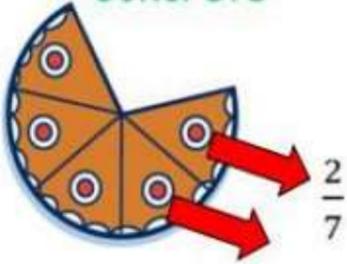
$\frac{2}{7}$

Pictorial
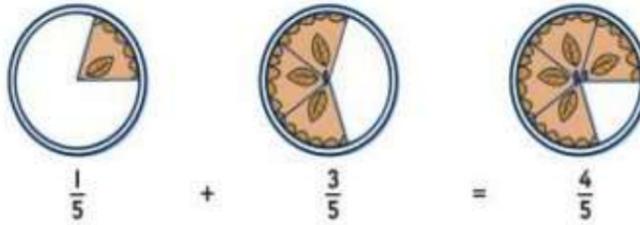
$\frac{1}{5}$ + $\frac{3}{5}$ = $\frac{4}{5}$

Abstract

Sam eats $\frac{2}{7}$ of a whole pizza. How much does he have left?

Lucy and Ben both eat $\frac{3}{8}$ of a cake. How much have they eat altogether?

35

Solve problems involving increasingly harder fractions to calculate quantities, and fractions to divide quantities, including non-unit fractions where the answer is a whole number.

Year 4

Fractions

Concrete

$\frac{2}{3}$

and decimal places.

Pictorial

$\frac{2}{3}$

$\frac{1}{3}$

Solve sim-

money problems involving fractions and decimals to
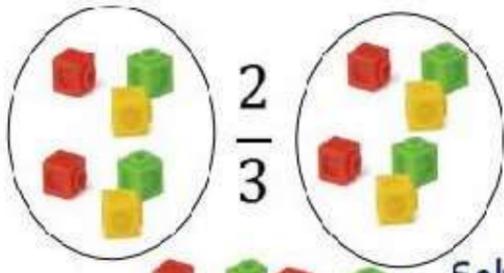
ple meas-

Abstract

$\frac{2}{3}$ of £18

£18 ÷ 3 = £6

£6 × 2 = £12

ure

two

Concrete

Pictorial

U	.	t	h
Units	Decimal Point	Tenths	Hundredths

Abstract

100cm = 1m

50cm = $\frac{1}{2}$ = 0.5m

25cm = $\frac{1}{4}$ = 0.25m

10cm = $\frac{1}{10}$ = 0.1m

30cm = $\frac{3}{10}$ = 0.3m

36

Year 5

Fractions

Identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths.

Concrete	Pictorial	Abstract
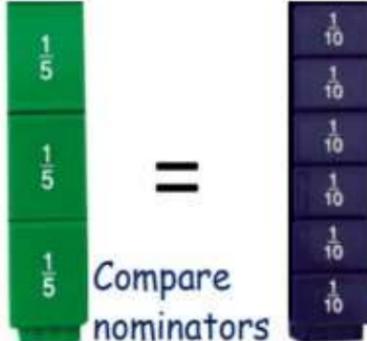
Compare de- nominators same number.

and order are all multi-

$$\frac{6}{10} = \frac{60}{100}$$ fractions whose ples of the

$$\frac{3}{5} = \frac{6}{10} = \frac{60}{100}$$

$$\frac{3}{4} = \frac{75}{100}$$

$$\frac{1}{5} = \frac{2}{10} = \frac{20}{100}$$

Concrete	Pictorial	Abstract
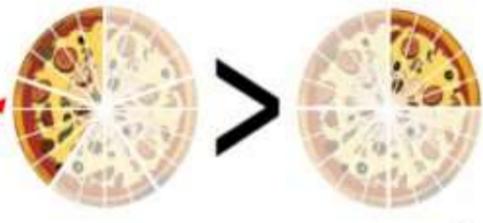
$$\frac{2}{5}$$
has become
$$\frac{8}{20}$$

$$\frac{1}{4}$$
has become
$$\frac{5}{20}$$

×4

$$\frac{2}{5} = \frac{8}{20} > \frac{1}{4} = \frac{5}{20}$$

×4   ×5

×5
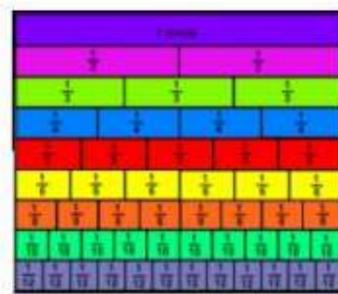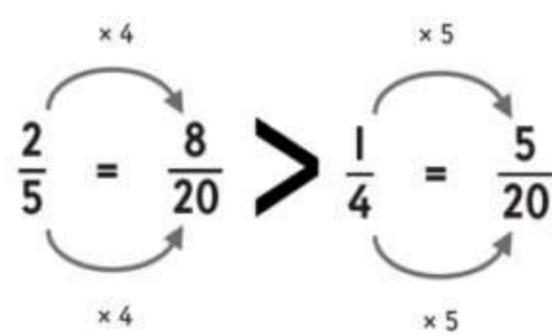
Recognise mixed numbers and improper fractions. Convert from one form to the other and write mathematical statements >1 as a mixed number.

Concrete	Pictorial	Abstract

Year 5
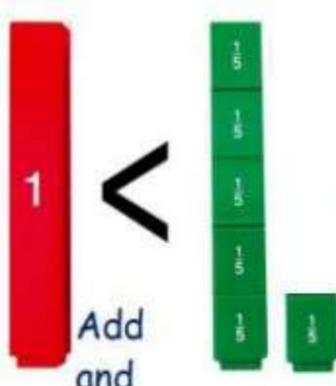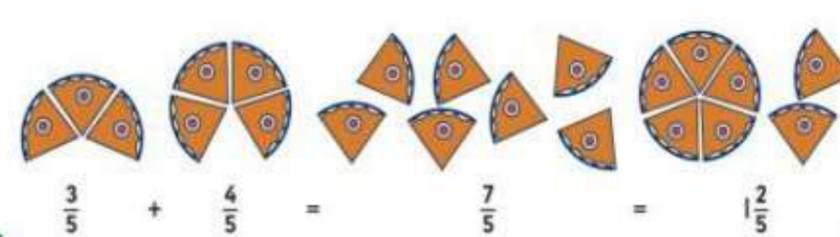
$$\frac{7}{2} = 3\frac{1}{2}$$
because 7 ÷ 2 = 3 with 1 half left over

$$2\frac{1}{3} = \frac{7}{3}$$
because 2 x 3 = 6 with 1 third left to add

$$\frac{3}{5} + \frac{4}{5} = \frac{7}{5} = 1\frac{2}{5}$$

Fractions

Add and subtract fractions with the same denominators and denominators that are multiples of the same numbers.

Concrete	Pictorial	Abstract

$$\frac{2}{5} - \frac{1}{4}$$

$$\frac{2}{5} = \frac{8}{20} \quad (\times 4)$$

$$\frac{1}{4} = \frac{5}{20} \quad (\times 5)$$

$$\frac{8}{20} - \frac{5}{20} = \frac{3}{20}$$

$$\text{So,} \quad \frac{2}{5} - \frac{1}{4} = \frac{3}{20}$$

$$\frac{8}{20} + \frac{5}{20} = \frac{13}{20}$$

$$\text{So,} \quad \frac{2}{5} + \frac{1}{4} = \frac{13}{20}$$

38

Year 5
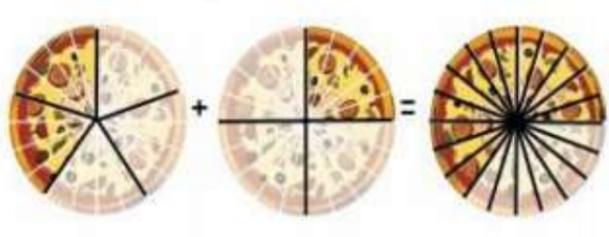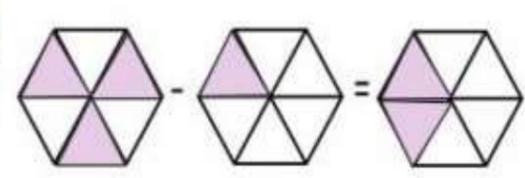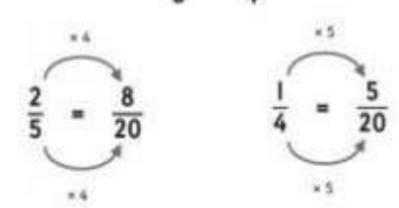
Fractions

Multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams.

Concrete	Pictorial	Abstract
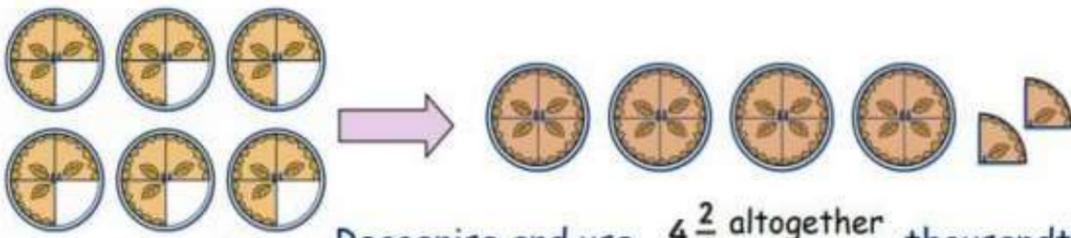
Multiply a proper fraction by a whole numb

$$\frac{3}{4} \times 6 = \frac{18}{4}$$

Change to a mixed number:

$$\frac{18}{4} = 4\frac{2}{4}$$

to   6 lots of $\frac{3}{4}$

$4\frac{2}{4}$ altogether

Recognise and use tenths, hundredths and decimal equivalents.

thousandths and relate them

Concrete	Pictorial	Abstract

67.153

How many thousandths does this number have? How many more thousandths do you need to add to make 67.16?

Th H T U

1    $\frac{1}{10}$ $\frac{1}{100}$ $\frac{1}{1000}$

Recog-   nise   %   sym-

bol and understand the meaning: write % as a fraction, decimal and percentage.

Concrete	Pictorial	Abstract

$\frac{3}{4}$ 0.75   75%

SALE

$^{40}/_{100}$ 0.40   40%
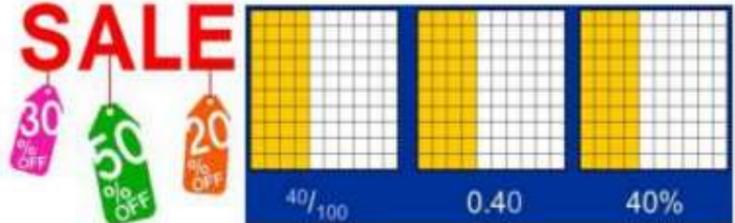
$\frac{4}{10} = 40\% = 0.4$

$\frac{32}{100} = 32\% = 0.32$

$\frac{75}{100} = 75\% = 0.75$

$\frac{2}{25} = \frac{8}{100} = 8\% = 0.08$

Year 6

Fractions

Add and subtract fractions with different denominators and mixed numbers using the concept of equivalent fractions.

Concrete

$1\frac{1}{2}$ Com-tions $\frac{1}{3}$ including $1\frac{5}{6}$

$1\frac{1}{2}$ **Pi** $\frac{9}{6}$

pare fractions >1.

$\frac{1}{3}$ $\frac{2}{6}$

and order

frac-

Abstract

$$1\frac{1}{2} + \frac{1}{3} = 1\frac{5}{6}$$

because $1\frac{1}{2} = \frac{3}{2}$

$\frac{3}{2} = \frac{9}{6}$ and $\frac{1}{3} = \frac{2}{6}$

so $\frac{9}{6} + \frac{2}{6} = \frac{11}{6} = 1\frac{5}{6}$

Concrete

Pictorial

Abstract

Which is greater?

$$\frac{2}{8} < \frac{6}{16}$$

Ordering from smallest to largest by using equivalent fractions

$$\frac{5}{12}, \frac{2}{3}, \frac{5}{6}$$

$$\frac{5}{12}, \frac{8}{12}, \frac{10}{12}$$

Use common factors to simplify fractions; use common multiples to express fractions in the same denomination.

Year 6

Fractions

Concrete

Mul-tiply simple proper fracing the answer in its simplest form.

Pictorial
$\frac{1}{3}$
$\frac{2}{6}$
$\frac{4}{12}$
pairs of tions, writ-

Abstract
$$\frac{18}{36} = \frac{6}{12} = \frac{1}{2}$$
$\div 3 \quad \div 6$

Concrete
$\frac{1}{2}$ of $\frac{3}{4}$

Pictorial
$\frac{1}{2}$ of $\frac{3}{4}$

Abstract
$$\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$$

1 multiply the numerators

$$\frac{2}{5} \times \frac{5}{6} = \frac{10}{30} = \frac{1}{3}$$ ← 3 simplify

2 multiply the denominators

41

Recall and use equivalences between simple fractions, decimals and percentages including in different contexts.

Year 6

Fractions

Concrete

Pictorial

Which would you prefer 75% or $\frac{3}{8}$ of a pie?

75% $\frac{3}{8}$ fractions

Divide   proper
by whole numbers.

Abstract

John scored $\frac{40}{80}$ in his spelling test and Hannah scored 40%. Who scored more?

John = $_{40}$ = **50%**

Hannah $\frac{}{}$ = **40%**

One paving slab is 0.3m long and another is $\frac{1}{4}$ of a metre. Which is longer?

$\frac{1}{4}$ = 0.25m

0.3m is larger than 0.25m

Concrete

Pictorial

$\frac{1}{2} \div 3 = \frac{1}{6}$

Abstract

$\frac{1}{2} \div 3 = \frac{1}{6}$

Keep it, change it, flip it!

$\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$

42

Associate fractions with division and calculate decimal fraction equivalents.

Concrete

0.25   0.25

0.25   0.25

quarters
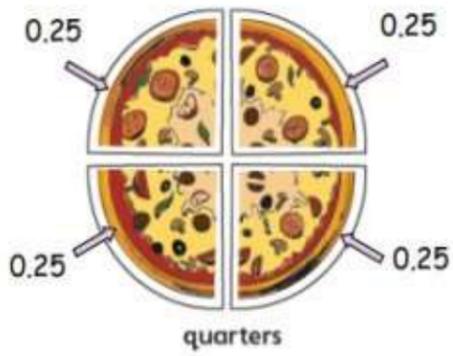
Pictorial

3 slices of pie 'out of' 8
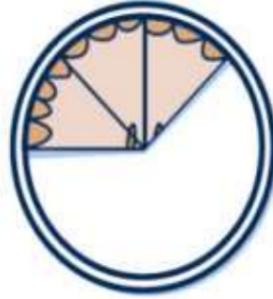
$\frac{3}{8}$

Abstract

$\frac{3}{8}$

3 'out of' 8 is the same as 3 'divided by' 8

$3 \div 8 = 0.375$

So $\frac{3}{8} = 0.375$

Year 6

Fractions

43